Names ____________________ Date ______

Math 10120 Exam 3 Solutions, Fall 2016

1. The following frequency table shows the number of pets owned by 31 students at Notre Dame. Find the **median**, M, for the set of data.

$x = \#Pets$	Frequency
0	10
1	5
2	7
3	3
4	1
5	3
10	2

Solution: Since there are 31 data points, the median will be the 16th number on the list, which is 2.

2. The Temperature readings at Noon in the town of Wagga Wagga for a random sample of 20 days are given in the following table:

Temperature	Frequency
60	2
61	1
62	5
65	9
67	2
70	1

The average temperature in the sample is $\bar{x} = 64$ (there is no need to check this). What is the sample standard deviation ?

Solution: We have

$$\begin{aligned} s^2 &= \frac{2(60-64)^2 + 1(61-64)^2 + 5(62-64)^2 + 9(65-64)^2 + 2(67-64)^2 + 1(70-64)^2}{19} \\ &= \frac{2(16) + 1(9) + 5(4) + 9(1) + 2(9) + 1(36)}{19} \\ &= \frac{124}{19}, \end{aligned}$$

so $s = \sqrt{\frac{124}{19}} \approx 2.55$.

3. A box contains four cards.

   - One card has the number 1 written on it,
   - two cards have the number 2 written on them and
   - one has the number 5 written on it.

   An experiment consists of drawing a sample of two cards from the box. Let X denote the sum of the numbers that appear on the cards drawn. Which of the following gives the probability distribution of the random variable X?

   Solution: The possible values of X are $1+2=3$, $2+2=4$, $1+5=6$, and $2+5=7$.

   - $P(X=3) = \frac{C(1,1)C(2,1)}{C(4,2)} = \frac{2}{6}$;
   - $P(X=4) = \frac{C(2,2)}{C(4,2)} = \frac{1}{6}$;
   - $P(X=6) = \frac{C(1,1)C(1,1)}{C(4,2)} = \frac{1}{6}$;
   - $P(X=7) = \frac{C(2,1)C(1,1)}{C(4,2)} = \frac{2}{6}$.

4. At a carnival game, the player plays \$1 to play and then rolls a pair of fair six-sided dice. The outcomes are:

$$\begin{matrix} \{(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\} \end{matrix}$$

   If the sum of the numbers on the uppermost face of the dice is 9 or higher, the game attendant gives the player \$A. Otherwise, the player receives nothing from the attendant. Let X denote the net earnings for the player for this game. Find the value of A which makes the game fair (i.e. $E(X) = 0$).

   Solution: There are exactly 10 possible outcomes in which the sum of the numbers on the dice is 9 or higher. Thus X has the following probability distribution:

X	P(X)
-1	$26/36$
$-1+A$	$10/36$

To find the value of A which makes the game fair, we set $E(X) = 0$ and solve for A:

$$\begin{aligned} E(X) &= (-1)\frac{26}{36} + (-1+A)\frac{10}{36} \\ 0 &= -\frac{36}{36} + \frac{10A}{36} \\ 36 &= 10A \\ 3.6 &= A. \end{aligned}$$

5. The probability distribution of a random variable X given below, find $E(X)$.

k	Pr(X = k)
-1	$\frac{1}{2}$
0	$\frac{1}{8}$
1	$\frac{1}{16}$
2	$\frac{1}{16}$
3	$\frac{1}{4}$

Solution:

$$E(X) = -1\left(\frac{1}{2}\right) + 0\left(\frac{1}{8}\right) + 1\left(\frac{1}{16}\right) + 2\left(\frac{1}{16}\right) + 3\left(\frac{1}{4}\right) = \frac{7}{16}.$$

6. A random variable X has the following probability distribution:

X	P(X)
−10	1/10
0	1/10
1	6/10
2	2/10

The expected value is $E(X) = 0$ (you don't need to check this). Which of the following gives the standard deviation of X?

Solution:

$$\begin{aligned}\sigma^2(X) &= \frac{1}{10}(-10-0)^2 + \frac{1}{10}(0-0)^2 + \frac{6}{10}(1-0)^2 + \frac{2}{10}(2-0)^2 \\ &= \frac{1}{10}(100) + 0 + \frac{6}{10}(1) + \frac{2}{10}(4) \\ &= \frac{114}{10},\end{aligned}$$

so $\sigma(X) = \sqrt{\frac{114}{10}}$.

7. An archer has a probability of 0.7 of hitting the center of the target on each attempt. Let X denote the number of times she will hit the target in her next 500 attempts. Assuming her performance on each attempt is independent of her performance on previous attempts, what is the expected value ($E(X)$) and standard deviation ($\sigma(X)$) of X?
Note: All numbers are rounded to two decimal places.

Solution: Since the result of each attempt is independent of previous attempts, X has a binomial distribution with $n = 500$, probability of success $p = 0.7$ and probability of failure $q = 0.3$. Thus $E(X) = np = 500(0.7) = 350$ and $\sigma(X) = \sqrt{npq} = \sqrt{500(.7)(.3)} \approx 10.25$.

8. A fair four sided die (with sides labelled 1-4) is rolled 5 times, what is the probability of getting at least two fours?

Solution: Let X be the number of 4s that occur in 5 rolls. Then X has a binomial distribution with $n = 5$, $p = \frac{1}{4}$ and $q = \frac{3}{4}$. We compute

$$\begin{aligned}P(X \geq 2) &= 1 - P(X < 2) \\ &= 1 - [P(X = 0) + P(X = 1)] \\ &= 1 - \left[C(5,0)\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 + C(5,1)\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4\right].\end{aligned}$$

9. If Z is a standard normal random variable, what is $P(-0.5 \leq Z \leq 1.5)$.

Solution: $P(-0.5 \leq Z \leq 1.5) = P(Z \leq 1.5) - P(Z \leq -0.5)$
$= A(1.5) - A(-0.5)$(from tables) $= .9332 - .3085 = .6247$.

10. The time a customer spends waiting to be seated at Mama Vino's restaurant on Friday night is normally distributed with mean 10 minutes and standard deviation 2.5 minutes. If you go to Mama Vino's next Friday night, what is the probability that you will have to wait longer than 13 minutes to be seated?

Solution: Let X be the number of minutes a customer spends waiting.

$$\begin{aligned} P(X > 13) &= P\left(\frac{X-10}{2.5} > \frac{13-10}{2.5}\right) \\ &= P(Z > 1.2) \\ &= 1 - P(Z \leq 1.2) \\ &= 1 - A(1.2) \\ &= 1 - 0.8849 \\ &= 0.1151. \end{aligned}$$

11. At a fairground game stand, you pay \$1 to play. You then throw 3 darts at a target from a distance of 20 feet. When you are done, the game attendant gives you \$2 for every time you hit the target. Let X denote the (net) earnings for the player for one play of this game. Suppose your probability of hitting the target on each throw is .4 and the outcomes on throws are independent of each other.

(a) Use the binomial probability distribution to write out the probability distribution for X when you are the player.

Solution: If you hit the target 0 times, then $X = -1$. If you hit 1 time, then $X = -1+2 = 1$. If you hit twice, $X = -1+2+2 = 3$. Finally, if you hit the target all three times, then $X = -1+2+2+2 = 5$.

X	Pr(X)
-1	$C(3,0)(0.4)^0(0.6)^3 = 0.216$
1	$C(3,1)(0.4)^1(0.6)^2 = 0.432$
3	$C(3,2)(0.4)^2(0.6)^1 = 0.288$
5	$C(3,3)(0.4)^3(0.6)^0 = 0.064$

(b) What is $E(X)$ (the expected value of X) for the probability distribution you found in Part (a)?

Solution:

$$E(X) = -1(0.216) + 1(0.432) + 3(0.288) + 5(0.064) = 1.4.$$

(c) If you played this game 30 times, how much would you expect to win? (assuming your chances of hitting the target remain the same throughout).

Solution: You would expect to win about $30 \cdot E(X) = 30(1.4) = 42$ dollars.

12. Melinda McNulty is running for election in Mathland. In the final two weeks of the election campaign her campaign managers will devote \$x to running local TV adds in Region X and \$y to running TV ads in Region Y. The campaign statistician tells the managers to expect to attract an average of 0.6 new voters (in favor of Melinda) per dollar spent on local TV ads in Region X and to attract an average of 0.4 new voters (in favor of Melinda) per dollar spent on local TV ads in Region Y.

(a) Melinda needs the sum of new voters from both regions to be at least 20,000. Express this constraint as an inequality in the variables x and y:

Solution: The expected number of new voters from Region X is $0.6x$; the expected number of new voters from Region Y is $0.4y$. Thus, to gain at least 20,000 new voters, we need

$$0.6x + 0.4y \geq 20,000.$$

(b) If the campaign managers spend \$20,000 on TV ads in Region X and \$10,000 on TV ads in Region Y, will they attract enough new voters to meet their requirements?

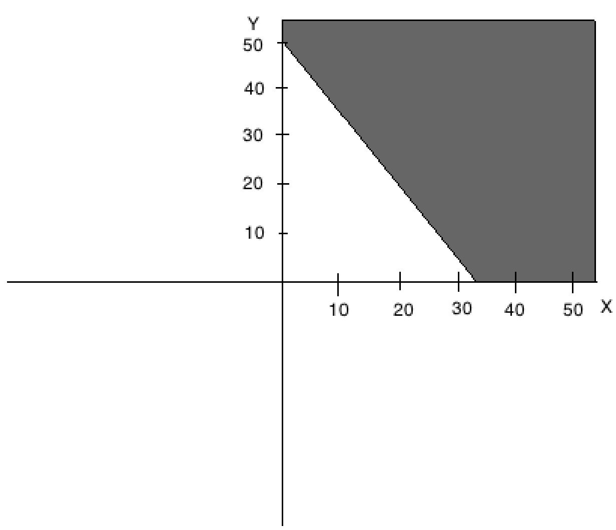
Solution:

$$0.6(20,000) + 0.4(10,000) = 16,000.$$

This is not enough to meet their requirements.

(c) Graph the inequality from part a) along with the inequalities $x \geq 0$ and $y \geq 0$.

Solution: The x-intercept for the line $0.6x + 0.4y = 20,000$ is $\approx 33,333$; the y-intercept is $50,000$. Thus, the graph of the inequality should look roughly like the one drawn below, where the units for the axes are in thousands:



13. Joe took admissions tests for two different apprenticeship programs. Test scores were normally distributed for both exams. The table below shows the mean and standard deviations for both exams along with Joe's score on each.

	Exam 1	Exam 2
Mean (μ)	72	55
Standard Deviation (σ)	8	12
Joe's Score	84	76

(a) Compute Joe's z-score on Exam 1.

Solution:

$$z = \frac{x - \mu}{\sigma} = \frac{84 - 72}{8} = 1.5.$$

(b) Compute Joe's z-score on Exam 2.

Solution:

$$z = \frac{x - \mu}{\sigma} = \frac{76 - 55}{12} = 1.75.$$

(c) On which test did Joe perform better relative to the other test takers?

Solution: Since Joe's z-score on the second exam is higher, he performed better on that one.

(d) What percentage of students who took Exam 1 had a score less than or equal to Joe's score of 84?

Solution:

$$P(X \leq 84) = P(Z \leq 1.5) = A(1.5) = 0.9332,$$

so Joe scored better than about 93.32% of the other students on Exam 1.

(e) What percentage of students who took Exam 2 had a score greater than equal to Joe's score of 76?

Solution:

$$P(X \geq 76) = P(Z \geq 1.75) = 1 - P(Z \leq 1.75) = 1 - A(1.75) = 1 - 0.9599 = 0.0401,$$

so about 4.01% of the students scored better than Joe on Exam 2.